International Journal of Scientific & Engineering Research Volume 9, Issue 5, May-2018
ISSN 2229-5518


# (*k, d*)- Odd Edge Mean Labeling of Some Trees

S. Dineshkumar, Dr.C. Dhevaki, Dr.K. Amuthavalli

**Abstract**— A $(p,q)$ graph G is said to have a (*k, d*)- odd edge mean labeling $(k,\ d \geq 1)$, if there exists an injection *f* from the edges of G to $\{0, 1, 2, 3, \ldots, 2k+2d(p-1)-1\}$ such that the induced map $f^*$ defined on V by $f^*(v)=\left\lceil \frac{\sum f(vu)}{\deg(v)} \right\rceil$ is a bijection from V to $\{2k-1, 2k+2d-1, 2k+4d-1, \ldots, 2k+2(p-1)d-1\}$. A graph that admits a (*k, d*)-odd edge mean labeling is called a (*k, d*)-odd edge mean graph. In this paper, we have introduced (*k, d*)-odd edge mean labeling and we have investigated the graphs like path, comb, twig, star and $P_m \odot nk_1$ graphs.

**Index Terms**— (*k, d*)- Odd edge mean labeling, (*k, d*)-Odd edge mean graph

## 1 Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [6]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges). Then the labeling is called a vertex labeling (or an edge labeling). Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [7]. Gayathri and Amuthavalli [1] extended this concept to *k*-odd mean labeling and (*k, d*) – odd mean labeling graphs.

In this paper, we have introduced the concept of (*k, d*) – odd edge mean labeling graphs.

For brevity, we use (*k, d*) – OEML for (*k, d*) – odd edge mean labeling and (*k, d*) – OEMG for (*k, d*) – odd edge mean graph.

## 2 Definitions

### 2.1 Definition

A $(p,q)$ graph G is said to have a (*k, d*)- odd edge mean labeling $(k \geq 1)$, if there exists an injection *f* from the edges of G to $\{0, 1, 2, 3, \ldots, 2k+2(p-1)d-1\}$ such that the induced map $f^*$ defined on V by $f^*(v)=\left\lceil \frac{\sum f(vu)}{\deg(v)} \right\rceil$ is a bijection from V to $\{2k-1, 2k+2d-1, 2k+4d-1, \ldots, 2k+2(p-1)d-1\}$. A graph that admits a (*k, d*)-odd edge mean labeling is called a (*k, d*)-odd edge mean graph.

### 2.2 Definition

A comb graph $P_n^+$ is a tree obtained from a path by attaching exactly one pendent edge to each vertex of the path.

### 2.3 Definition

A twig is a tree obtained from a path by attaching exactly two pendent edges to each internal vertex of the path..

## 3 Main Results

**Theorem 3.1** *The path graph* $P_n$ *is a (k, d)-odd edge mean graph for all k and d, when n is odd.*

**Proof.** Let $V(P_n)=\{v_i, 1 \leq i \leq n\}$ and $E(P_n)=\{e_i, 1 \leq i \leq n-1\}$. (See Fig 3.1)

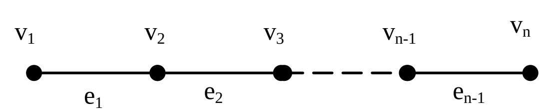



Fig 3.1

First we label the edges as follows:

Define $f: E \to \{0, 1, 2, 3, \ldots, 2k+2(p-1)d-1\}$ by

$$f(e_1)=2k-1$$

* *Research Scholar, PG and Research Department of Mathematics, Government Arts College, Ariyalur, Tamilnadu – 621704.* Email-kingsdina@gmail.com
* *PG Assistant( Mathematics) Government Hr. Sec School, Anukkur, Veppanthattai, Perambalur, Tamil nadu – 621219.* Email-dhevakirajarajan@gmail.com
* *Assistant Professor, Department of Mathematics, Government Arts and Science College, Veppanthattai,Perambalur, Tamilnadu – 621116.* Email-thrcka@gmail.com

Case (i): n=3

$$f(e_2) = 2k + 4d - 1$$

Case (ii): n >3

For $2 \le i \le n-3$,

$$f(e_i) = \begin{cases} 2k + 2d(i-1), & i \text{ is odd} \\ 2k + 2di - 2, & i \text{ is even} \end{cases}$$

$$f(e_{n-2}) = 2k + 2d(n-3) - 1,$$

$$f(e_{n-1}) = 2k + 2d(n-1) - 1$$

Then the induced vertex labels are

For $1 \le i \le n$,

$$f^*(v_i) = 2k + 2d(i-1) - 1$$

Thus

$$V(P_n) = \{2k-1,\ 2k+2d-1,\ \ldots,\ 2k+2d(p-1)-1\}$$

Hence, the path $P_n$ is a (*k*, *d*)-odd edge mean graph for all *k* and *d* when n is odd.

**Theorem 3.2** *The graph $P_n^+$ is a (k, d)-odd edge mean graph for all k and d when n ($n \ge 4$) is even.*

**Proof** Let $V(P_n^+) = \{u_1, u_2, u_3, \ldots u_n, v_1, v_2, v_3, \ldots v_n\}$

$$E(P_n^+) = \{u_i u_{i+1}, 1 \le i \le n-1\} \cup \{u_i v_i, 1 \le i \le n\}$$

(See Fig 3.2)

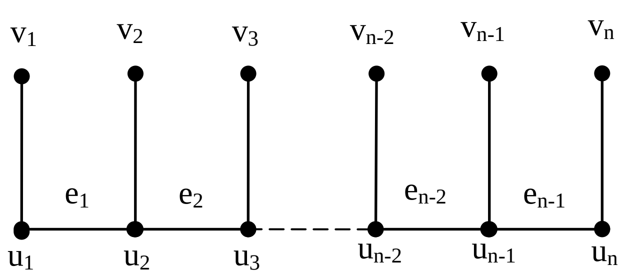



Fig 3.2

First we label the edges as follows:

Define $f : E \to \{0, 1, 2, 3, \ldots, 2k + 2(p-1)d - 1\}$ by

$$f(u_1 u_2) = 2k + 4d - 2$$

$$f(u_2 u_3) = 2k$$

Case (i) : n = 4

$$f(u_3 u_4) = 2k + 6d - 2,$$

Case (ii) : n >4

For $3 \le i \le n-3$,

$$f(u_i u_{i+1}) = 2k + 2d(2i-3) - 2,$$

$$f(u_{n-2} u_{n-1}) = 2k + 4d(n-4),$$

$$f(u_{n-1} u_n) = 2k + 2d(2n-5) - 2$$

$$f(u_1 v_1) = 2k - 1$$

For $2 \le i \le n-1$,

$$f(u_i v_i) = 2k + 4di - 1,$$

$$f(u_n v_n) = 2k + 2d(2n-1) - 1$$

Then the induced vertex labels are

$$f^*(u_1) = 2k + 2d - 1$$

$$f^*(u_2) = 2k + 4d - 1$$

For $3 \le i \le n$,

$$f^*(u_i) = 2k + 2d(2i-3) - 1,$$

$$f^*(v_1) = 2k - 1$$

For $2 \le i \le n-1$,

$$f^*(v_i) = 2k + 4di - 1,$$

$$f^*(v_n) = 2k + 2d(2n-1) - 1$$

Thus

$$V(P_n^+) = \{2k-1,\ 2k+2d-1,\ 2k+4d-1, \ldots,\ 2k+2d(p-1)-1\}$$

Hence, the graph $P_n^+$ is a (*k*, *d*)-odd edge mean graph for all k and d when n is even and $n \ge 4$.

**Theorem 3.3** *The twig graph $T_n$ is a (k, d)-odd edge mean graph for all k and d when n is odd.*

**Proof** Let $V(T_n) = \{v_i, 1 \le i \le n\} \cup \{u_i, w_i, 1 \le i \le n-2\}$ and

$$E(T_n) = \{v_{i+1} u_i,\ v_{i+1} w_i, 1 \le i \le n-2\} \cup \{v_i v_{i+1}, 1 \le i \le n-1\}.$$

(See Fig 3.3)

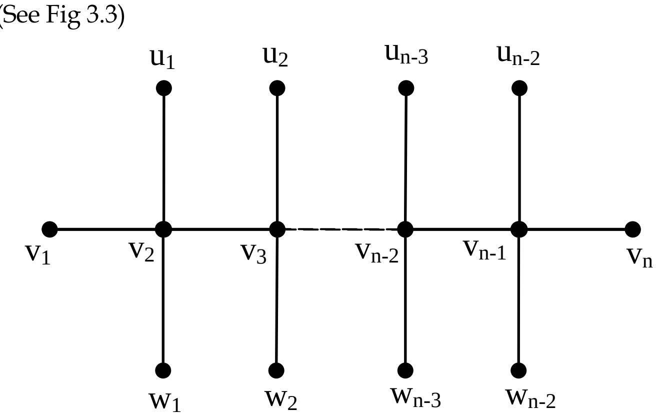



Fig 3.3

First we label the edges as follows:

Define $f : E \to \{0, 1, 2, 3, \ldots, 2k + 2(p-1)d - 1\}$ by

$$f(v_1 v_2) = 2k - 1$$

For $2 \le i \le n-2$,

$$f\left(v_i v_{i+1}\right)=\begin{cases}2k+6d(i-1)-2, & i \text{ is odd}\\ 2k+2d(3i-2)-2, & i \text{ is even}\end{cases}$$

$$f\left(v_{n-1}v_n\right)=2k+2d(3n-5)-1$$

For $1 \le i \le n-2$,

$$f\left(u_i v_{i+1}\right)=2k+2d(3i-2)-1,$$

$$f\left(w_i v_{i+1}\right)=2k+6di-1,$$

Then the induced vertex labels are

For $1 \le i \le n-2$,

$$f^*\left(u_i\right)=2k+2d(3i-2)-1,$$

$$f^*\left(v_1\right)=2k-1$$

For $2 \le i \le n-1$,

$$f^*\left(v_i\right)=2k+2d(3i-4)-1,$$

$$f^*\left(v_n\right)=2k+2d(3n-5)-1$$

For $1 \le i \le n-2$,

$$f^*\left(w_i\right)=2k+6di-1,$$

Thus

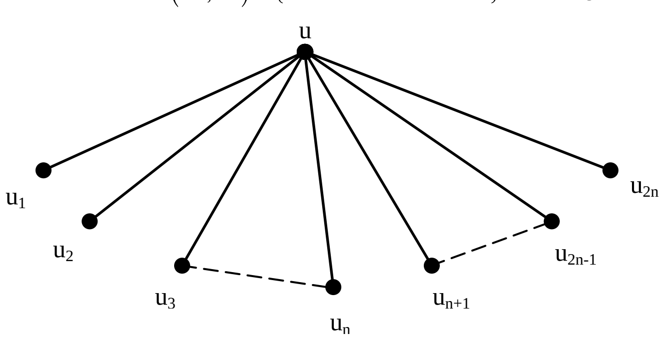
$$V(T_n)=\{2k-1,\ 2k+2d-1,\ 2k+4d-1, \ldots,\ 2k+2d(p-1)-1\}$$

Hence, the twig graph $T_n$ is a (*k, d*)-odd edge mean graph for all k and d when n is odd.

**Theorem 3.4** *The Star* $K_{1,2n}$ *is a (k, d)-odd edge mean graph for any k and d.*

**Proof** Let $V\left(K_{1,\,2n}\right)=\{u,\ u_1,\ u_2,\ u_3, \ldots,\ u_{2n}\}$ and

$E\left(K_{1,\,2n}\right)=\{e_i=uu_i,\ 1 \le i \le 2n\}$. See Fig 3.4




First we lal

Fig 3,4

Define $f: E \to \{0, 1, 2, 3, \ldots,\ 2k+2d(p-1)-1\}$ by

$$f\left(e_i\right)=\begin{cases}2k+2d(i-1)-1, & 1 \le i \le n\\ 2k+2di-1, & n+1 \le i \le 2n\end{cases}$$

Then the induced vertex labels are

$$f\left(u\right)=2k+2dn-1$$

$$f^*\left(u_i\right)=\begin{cases}2k+2d(i-1)-1, & \text{for } 1 \le i \le n\\ 2k+2di-1, & \text{for } n+1 \le i \le 2n\end{cases}$$

Thus

$$V(K_{1,2n})=\{2k-1,\ 2k+2d-1,\ 2k+4d-1, \ldots,\ 2k+2d(p-1)-1\}$$

Hence, the Star $K_{1,2n}$ is a (*k, d*)-odd edge mean graph for any k and d.

**Theorem 3.5** *The graph $P_m \,\Theta\, nk_1$ is a (k, d)-odd edge mean graph for all k and d when m is odd and n is even.*

**Proof** Let

$$V\left(P_m \Theta nk_1\right)=\{u_i,\ 1 \le i \le m\} \cup \{u_{ij},\ 1 \le i \le m, 1 \le j \le n\}$$

$$E\left(P_m \Theta nk_1\right)=\{e_i,\ 1 \le i \le m-1\} \cup \{e_{ij},\ 1 \le i \le m, 1 \le j \le n\}$$ (See Fig 3.5)

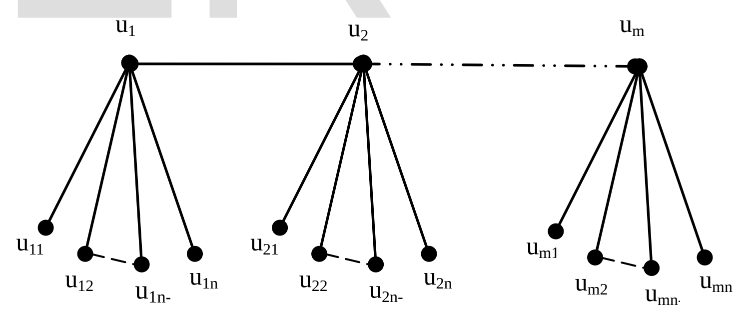



Fig 3.5

First we label the edges as follows:

Define $f: E \to \{0, 1, 2, 3, \ldots,\ 2k+2(p-1)d-1\}$ by

For $1 \le i \le m-2$,

$$f\left(e_i\right)=\begin{cases}2k+2di(n+1)-d(n+2)-1, & i \text{ is odd}\\ 2k+2di(n+1)+dn-2, & i \text{ is even}\end{cases}$$

$$f\left(e_{m-1}\right)=2k+2dm(n+1)-d(n+2)-1$$

For $1 \le i \le m$,

ISSN 2229-5518

$$f\left(e_{ij}\right)=\begin{cases}2k+2d[i+n(i-1)+j-2]-1, & 1\le j\le n/2\\ 2k+2d[i+n(i-1)+j-1]-1, & \frac{n}{2}+1\le j\le n\end{cases}$$

Then the induced vertex labels are

For $1 \le i \le m$,

$$f^*\left(u_i\right)=2k+dn(2i-1)+2d(i-1)-1,$$

$$f^*\left(u_{ij}\right)=\begin{cases}2k+2d[i+n(i-1)+j-2]-1, & 1\le j\le n/2\\ 2k+2d[i+n(i-1)+j-1]-1, & \frac{n}{2}+1\le j\le n\end{cases}$$

Thus

$$V(P_m \Theta nk_1)=\left\{2k-1,\ 2k+2d-1,\ \ldots,\ 2k+2d(p-1)-1\right\}$$

Hence, the graph $P_m \Theta nk_1$ is a (k, d)-odd edge mean graph for all k and d when m is odd and n is even.

## REFERENCES

[1] K.Amuthavalli, "Graph labeling and its applications - some generalizations of odd mean labeling", *Ph.D. thesis*, Mother Theresa women's university, Kodaikanal, July (2010).

[2] K.Amuthavalli, and S.Dineshkumar, "k- Odd edge mean labeling of Some Basic graphs", *Journal of Informatics and Mathematical Science*, Vol. 9, No. 3, ISSN 0975-5748 (online); 0974-875X (print) Published by RGN Publications (Publication Process).

[3] J.A. Gallian, "A dynamic survey of graph labeling", *Electronic Journal of Combinatorics*, 18 (2014) # DS6.

[4] B.Gayathri and K.Amuthavalli, "k-odd mean labeling of crown graphs", *International Journal of Mathematics and Computer Science*, 2(3) (2007) 253-259.

[5] B.Gayathri and K.Amuthavalli, "(k,d)-odd mean labeling of some graphs", *Bulletin of Pure and Applied Sciences*, 26E (2) (2007) 263-267.

[6] F.Harary, "Graph Theory", *Addision –Wesley*, Reading Masaachusetts, 1972.

[7] Rosa, "On certain valuations of the vertices of a graph Theory of Graphs (Internet Symposium, Rome, July 1966)", Gordon and Breach, N.Y. and Duhod, Paris (1967) 349-355.